LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **JULY 2025**

UMT 6501 – COMPLEX ANALYSIS

Date: 11-07-2025 Dept. No. Max. : 100 Marks

Time: 10:00 AM - 01:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	Answer the following
a)	Find the real and imaginary parts of $\frac{4}{6-7i}$
b)	State the relationship between differentiable and continuous functions
c)	Check whether $u=5x^4-5y^4$ is harmonic or not
d)	Find the isolated singular points of the function $\frac{z+1}{z^3(z^2+1)}$
e)	Define residue of a function at a point
2.	Fill in the blanks
a)	$A\text{rg}(-i)=----$
b)	If $f'(z)=0$ in a domain D, then $f(z)$ must be a -----------throughout D.
c)	Any polynomial of degree $n(n\geq 1)$ has at least one ---------
d)	The residue of $\frac{z^3+2z}{(z-i)^3}$ at $z=i$ is ------------
e)	The transformation $w=\bar{z}$ is isogonal but not -------------
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	$\lim\limits_{z\to -1}\frac{iz+3}{z+1}=0$
b)	Laplace equation is $H_{xx}(x,y)+H_{yy}(x,y)=1$.
c)	The sequence $z_n=\frac{1}{n^3}+i(n=1,2,3,...)$ converges to i
d)	To evaluate integrals of the type $\int_0^{2\pi} f(\cos\theta,\sin\theta)d\theta$ Cauchy residue theorem may be used
e)	The transformation $w=\frac{az+b}{cz+d}(ad-bc\neq 0)$ where a,b,c,d are complex constants is called linear fractional transformation
4.	**Answer the following** MCQ
a)	If $z=3+i$, then $\lvert z\rvert=$ (i) 10 (ii) 20 (iii) 14 (iv) 12

b)	$\int_0^1 (1+it)^2\,dt =$ -------- (i) 2 (ii)3 (iii) $\frac{2}{3}+i$ (iv) $\frac{2}{3}-i$
c)	If a series of complex numbers converges, the nth term converges to -------- as n tends to infinity. (i) 1 (ii) 0 (iii) 2 (iv) 3
d)	The critical points of $z+\frac{1}{z}$ are (i) 1.-1, (ii) 1,0 (iii)1,1 (iv)1,2
e)	A singular point of the function $\frac{1}{z(z-i)}$ is (i) 0 (ii) 1 (iii) 2 (iv)3
SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	List all the values of cubic root of -1
6.	Verify Cauchy – Riemann equations for $f(z)=\sin z$
7.	Find the Laurent's series for $e^{\frac{1}{z}}$ about $z=0$
8.	Discuss any 4 transformations of $w=\frac{1}{z}$
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Derive Cauchy Riemann equations in polar coordinates for a differentiable function
10.	If a function $f(z)=u+iv$ is anlaytic in a domain D, Establish that its component functions are harmonic in D.
11.	State and prove Cauchy's residue theorem and hence evaluate $\int_C \frac{5z-2}{z(z-1)}dz$ where C is the circle $\|z\|=2.$
12.	Define singularity and discuss its types
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	Prove that if $f(z)=u+iv$ is differentiable at a point $z_0=x_0+iy_0$ then u and v have first order partial derivatives at (x_0,y_0) and these partial derivatives satisfy the Cauchy Riemann equations.
14.	State and prove Taylor's Theorem and obtain the Taylor's series expansion of $\frac{1}{1-z}$
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	Derive Cauchy integral formula and hence evaluate $\int_C \frac{z}{(9-z^2)}dz$ where C is the circle $\|z\|=3$
16.	Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4sin\theta}$.

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